

**Sample Question Paper**  
**CLASS: XII**  
**Session: 2021-22**  
**Mathematics (Code-041)**  
**Term - 2**

Time Allowed: 2 hours

Maximum Marks: 40

**General Instructions:**


1. This question paper contains **three sections - A, B and C**. Each part is compulsory.
2. **Section - A** has 6 **short answer type (SA1)** questions of 2 marks each.
3. **Section - B** has 4 **short answer type (SA2)** questions of 3 marks each.
4. **Section - C** has 4 **long answer type questions (LA)** of 4 marks each.
5. There is an **internal choice** in some of the questions.
6. Q14 is a **case-based problem** having 2 sub parts of 2 marks each.

**SECTION - A**

1.	Find $\int \frac{\log x}{(1+\log x)^2} dx$  OR Find $\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}} dx$	2
2.	Write the sum of the order and the degree of the following differential equation: $\frac{d}{dx} \left( \frac{dy}{dx} \right) = 5$	2
3.	If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that $ \hat{a} + \hat{b}  = 2\cos \frac{\theta}{2}$ , where $\theta$ is the angle between them.	2
4.	Find the direction cosines of the following line: $\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$	2
5.	A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement.	2
6.	Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack?	2

**SECTION - B**

7.	Find: $\int \frac{x+1}{(x^2+1)x} dx$	3
8.	Find the general solution of the following differential equation: $x \frac{dy}{dx} = y - x \sin\left(\frac{y}{x}\right)$  OR Find the particular solution of the following differential equation, given that $y = 0$ when $x = \frac{\pi}{4}$ : $\frac{dy}{dx} + y \cot x = \frac{2}{1 + \sin x}$	3
9.	If $\vec{a} \neq \vec{0}$ , $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ , $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ , then show that $\vec{b} = \vec{c}$ .	3

10.	Find the shortest distance between the following lines: $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + s(2\hat{i} + \hat{j} + \hat{k})$ $\vec{r} = (\hat{i} + \hat{j} + 2\hat{k}) + t(4\hat{i} + 2\hat{j} + 2\hat{k})$ OR Find the vector and the cartesian equations of the plane containing the point $\hat{i} + 2\hat{j} - \hat{k}$ and parallel to the lines $\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + s(2\hat{i} - 3\hat{j} + 2\hat{k}) = 0$ and $\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + t(\hat{i} - 3\hat{j} + \hat{k}) = 0$	3
<b><u>SECTION - C</u></b>		
11.	Evaluate: $\int_{-1}^2  x^3 - 3x^2 + 2x  dx$	4
12.	Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$ , the parabola $y^2 = x$ and the x-axis. OR Using integration, find the area of the region $\{(x, y): 0 \leq y \leq \sqrt{3}x, x^2 + y^2 \leq 4\}$	4
13.	Find the foot of the perpendicular from the point $(1, 2, 0)$ upon the plane $x - 3y + 2z = 9$ . Hence, find the distance of the point $(1, 2, 0)$ from the given plane.	4
14.	<p style="text-align: center;"><b><u>CASE-BASED/DATA-BASED</u></b></p>  <p>Fig 3</p> <p>An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.</p> <p><b>Based on the given information, answer the following questions.</b></p>	
	(i) what is the probability that a new policyholder will have an accident within a year of purchasing a policy?	2
	(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?	2

**Marking Scheme**  
**CLASS: XII**  
**Session: 2021-22**  
**Mathematics (Code-041)**  
**Term - 2**

**SECTION – A**

1.	<p>Find: <math>\int \frac{\log x}{(1+\log x)^2} dx</math></p> <p>Solution: <math>\int \frac{\log x}{(1+\log x)^2} dx = \int \frac{\log x + 1 - 1}{(1+\log x)^2} dx = \int \frac{1}{1+\log x} dx - \int \frac{1}{(1+\log x)^2} dx</math></p> <p><math>= \frac{1}{1+\log x} \times x - \int \frac{-1}{(1+\log x)^2} \times \frac{1}{x} \times x dx - \int \frac{1}{(1+\log x)^2} dx = \frac{x}{1+\log x} + c</math></p> <p style="text-align: center;">OR</p> <p>Find: <math>\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}} dx</math></p> <p>Solution: Put <math>\cos^2 x = t \Rightarrow -2\cos x \sin x dx = dt \Rightarrow \sin 2x dx = -dt</math></p> <p>The given integral <math>= - \int \frac{dt}{\sqrt{3^2-t^2}} = -\sin^{-1} \frac{t}{3} + c = -\sin^{-1} \frac{\cos^2 x}{3} + c</math></p>	<p>1/2</p> <p>1+1/2</p> <p>1</p> <p>1</p>
2.	<p>Write the sum of the order and the degree of the following differential equation: <math>\frac{d}{dx} \left( \frac{dy}{dx} \right) = 5</math></p> <p>Solution: Order = 2 Degree = 1 Sum = 3</p>	<p>1</p> <p>1/2</p> <p>1/2</p>
3.	<p>If <math>\hat{a}</math> and <math>\hat{b}</math> are unit vectors, then prove that <math> \hat{a} + \hat{b}  = 2\cos \frac{\theta}{2}</math>, where <math>\theta</math> is the angle between them.</p> <p>Solution: <math>(\hat{a} + \hat{b}) \cdot (\hat{a} + \hat{b}) =  \hat{a} ^2 +  \hat{b} ^2 + 2(\hat{a} \cdot \hat{b})</math></p> <p><math> \hat{a} + \hat{b} ^2 = 1 + 1 + 2\cos \theta</math></p> <p><math>= 2(1 + \cos \theta) = 4\cos^2 \frac{\theta}{2}</math></p> <p><math>\therefore  \hat{a} + \hat{b}  = 2\cos \frac{\theta}{2}</math></p>	<p>1</p> <p>1/2</p> <p>1/2</p>
4.	<p>Find the direction cosines of the following line:</p> <p><math>\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}</math></p> <p>Solution: The given line is</p> <p><math>\frac{x-3}{1} = \frac{y-\frac{1}{2}}{1} = \frac{z}{4}</math></p> <p>Its direction ratios are <math>\langle 1, 1, 4 \rangle</math></p> <p>Its direction cosines are</p> <p><math>\left\langle \frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}} \right\rangle</math></p>	<p>1</p> <p>1/2</p> <p>1/2</p>

5.	<p>A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement.</p> <p>Solution: Let X be the random variable defined as the number of red balls. Then <math>X = 0, 1</math></p> $P(X=0) = \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$ $P(X=1) = \frac{1}{4} \times \frac{3}{3} + \frac{3}{4} \times \frac{1}{3} = \frac{6}{12} = \frac{1}{2}$ <p>Probability Distribution Table:</p> <table border="1"> <tr> <td>X</td><td>0</td><td>1</td></tr> <tr> <td>P(X)</td><td><math>\frac{1}{2}</math></td><td><math>\frac{1}{2}</math></td></tr> </table>	X	0	1	P(X)	$\frac{1}{2}$	$\frac{1}{2}$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p><math>\frac{1}{2}</math></p>
X	0	1						
P(X)	$\frac{1}{2}$	$\frac{1}{2}$						
6.	<p>Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack?</p> <p>Solution: The required probability = P((The first is a red jack card and The second is a jack card) or (The first is a red non-jack card and The second is a jack card))</p> $= \frac{2}{52} \times \frac{3}{51} + \frac{24}{52} \times \frac{4}{51} = \frac{1}{26}$	<p>1</p> <p>1</p>						
<b><u>SECTION – B</u></b>								
7.	<p>Find: <math>\int \frac{x+1}{(x^2+1)x} dx</math></p> <p>Solution: Let <math>\frac{x+1}{(x^2+1)x} = \frac{Ax+B}{x^2+1} + \frac{C}{x} = \frac{(Ax+B)x+C(x^2+1)}{(x^2+1)x}</math></p> $\Rightarrow x+1 = (Ax+B)x + C(x^2+1) \quad (\text{An identity})$ <p>Equating the coefficients, we get</p> <p><math>B = 1, C = 1, A + C = 0</math></p> <p>Hence, <math>A = -1, B = 1, C = 1</math></p> <p>The given integral = <math>\int \frac{-x+1}{x^2+1} dx + \int \frac{1}{x} dx</math></p> $= \frac{-1}{2} \int \frac{2x-2}{x^2+1} dx + \int \frac{1}{x} dx$ $= \frac{-1}{2} \int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx + \int \frac{1}{x} dx$ $= \frac{-1}{2} \log(x^2+1) + \tan^{-1} x + \log x  + c$	<p>1/2</p> <p><math>\frac{1}{2}</math></p> <p>1/2</p> <p>1+1/2</p>						
8.	<p>Find the general solution of the following differential equation:</p> $x \frac{dy}{dx} = y - x \sin\left(\frac{y}{x}\right)$ <p>Solution: We have the differential equation:</p> $\frac{dy}{dx} = \frac{y}{x} - \sin\left(\frac{y}{x}\right)$ <p>The equation is a homogeneous differential equation.</p> <p>Putting <math>y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}</math></p> <p>The differential equation becomes</p> $v + x \frac{dv}{dx} = v - \sin v$ $\Rightarrow \frac{dv}{\sin v} = -\frac{dx}{x} \Rightarrow \operatorname{cosec} v dv = -\frac{dx}{x}$ <p>Integrating both sides, we get</p>	<p>1</p> <p><math>\frac{1}{2}</math></p>						



	<p>Solution: Here, the lines are parallel. The shortest distance = <math>\frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} }</math></p> $= \frac{ (3\hat{k}) \times (2\hat{i} + \hat{j} + \hat{k}) }{\sqrt{4 + 1 + 1}}$ $(3\hat{k}) \times (2\hat{i} + \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{i} + 6\hat{j}$ <p>Hence, the required shortest distance = <math>\frac{3\sqrt{5}}{\sqrt{6}}</math> units OR</p> <p>Find the vector and the cartesian equations of the plane containing the point <math>\hat{i} + 2\hat{j} - \hat{k}</math> and parallel to the lines <math>\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + s(2\hat{i} - 3\hat{j} + 2\hat{k}) = 0</math> and <math>\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + t(\hat{i} - 3\hat{j} + \hat{k}) = 0</math></p> <p>Solution: Since, the plane is parallel to the given lines, the cross product of the vectors <math>2\hat{i} - 3\hat{j} + 2\hat{k}</math> and <math>\hat{i} - 3\hat{j} + \hat{k}</math> will be a normal to the plane</p> $(2\hat{i} - 3\hat{j} + 2\hat{k}) \times (\hat{i} - 3\hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 1 & -3 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{k}$ <p>The vector equation of the plane is <math>\vec{r} \cdot (3\hat{i} - 3\hat{k}) = (\hat{i} + 2\hat{j} - \hat{k}) \cdot (3\hat{i} - 3\hat{k})</math> or, <math>\vec{r} \cdot (\hat{i} - \hat{k}) = 2</math> and the cartesian equation of the plane is <math>x - z - 2 = 0</math></p>	<p>1+1/2</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1</p>
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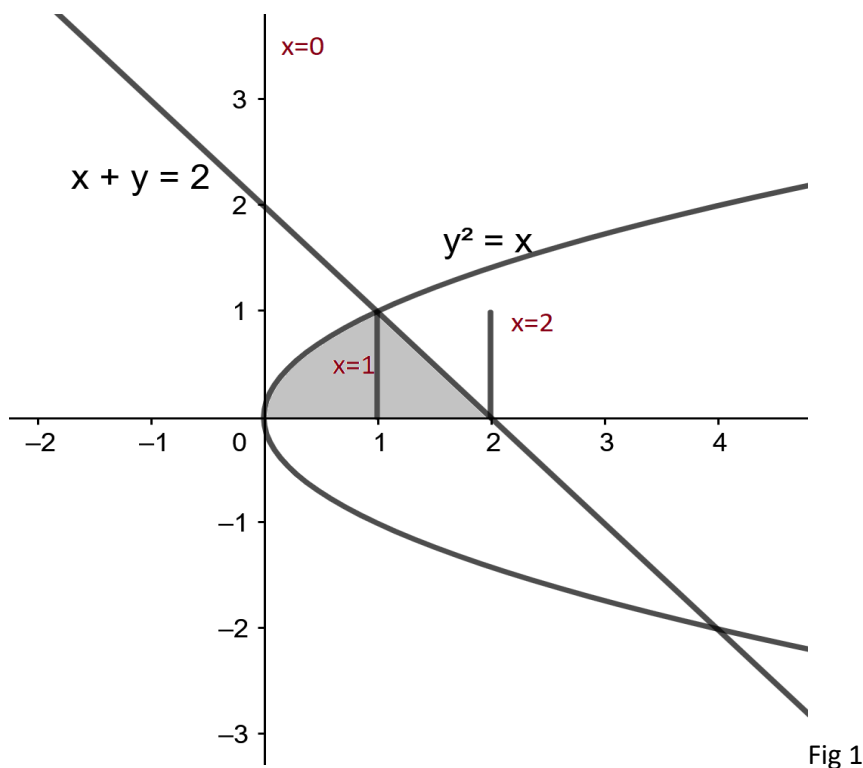
### SECTION – C

11.	<p>Evaluate: <math>\int_{-1}^2  x^3 - 3x^2 + 2x  dx</math></p> <p>Solution: The given definite integral = <math>\int_{-1}^2  x(x-1)(x-2)  dx</math></p> $= \int_{-1}^0  x(x-1)(x-2)  dx + \int_0^1  x(x-1)(x-2)  dx + \int_1^2  x(x-1)(x-2)  dx$ $= -\int_{-1}^0 (x^3 - 3x^2 + 2x) dx + \int_0^1 (x^3 - 3x^2 + 2x) dx - \int_1^2 (x^3 - 3x^2 + 2x) dx$ $= -\left[\frac{x^4}{4} - x^3 + x^2\right]_{-1}^0 + \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 - \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$ $= \frac{9}{4} + \frac{1}{4} + \frac{1}{4} = \frac{11}{4}$	<p>1+1/2</p> <p>1/2</p> <p>2</p>
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12. Using integration, find the area of the region in the first quadrant enclosed by the line  $x + y = 2$ , the parabola  $y^2 = x$  and the x-axis.  
 Solution: Solving  $x + y = 2$  and  $y^2 = x$  simultaneously, we get the points of intersection as  $(1, 1)$  and  $(4, -2)$ .

1



1

The required area = the shaded area =  $\int_0^1 \sqrt{x} dx + \int_1^2 (2 - x) dx$   
 $= \frac{2}{3} [x^{\frac{3}{2}}]_0^1 + [2x - \frac{x^2}{2}]_1^2$   
 $= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$  square units

1

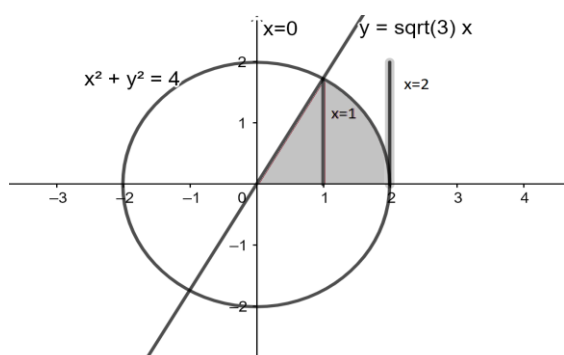
1

OR

Using integration, find the area of the region:  $\{(x, y): 0 \leq y \leq \sqrt{3}x, x^2 + y^2 \leq 4\}$

Solution: Solving  $y = \sqrt{3}x$  and  $x^2 + y^2 = 4$ , we get the points of intersection as  $(1, \sqrt{3})$  and  $(-1, -\sqrt{3})$


1



1

	<p>The required area = the shaded area = <math>\int_0^1 \sqrt{3}x \, dx + \int_1^2 \sqrt{4-x^2} \, dx</math></p> $= \frac{\sqrt{3}}{2} [x^2]_0^1 + \frac{1}{2} [x\sqrt{4-x^2} + 4 \sin^{-1} \frac{x}{2}]_1^2$ $= \frac{\sqrt{3}}{2} + \frac{1}{2} [2\pi - \sqrt{3} - 2 \frac{\pi}{3}]$ $= \frac{2\pi}{3} \text{ square units}$	<p>1</p> <p>1</p>
13.	<p>Find the foot of the perpendicular from the point (1, 2, 0) upon the plane <math>x - 3y + 2z = 9</math>. Hence, find the distance of the point (1, 2, 0) from the given plane.</p> <p>Solution: The equation of the line perpendicular to the plane and passing through the point (1, 2, 0) is</p> $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z}{2}$ <p>The coordinates of the foot of the perpendicular are <math>(\mu + 1, -3\mu + 2, 2\mu)</math> for some <math>\mu</math></p> <p>These coordinates will satisfy the equation of the plane. Hence, we have</p> $\mu + 1 - 3(-3\mu + 2) + 2(2\mu) = 9$ $\Rightarrow \mu = 1$ <p>The foot of the perpendicular is (2, -1, 2).</p> <p>Hence, the required distance = <math>\sqrt{(1-2)^2 + (2+1)^2 + (0-2)^2} = \sqrt{14} \text{ units}</math></p>	<p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p>



14.	<p style="text-align: center;"><b><u>CASE-BASED/DATA-BASED</u></b></p> <div style="text-align: center;">  </div> <p><b>Fig 3</b></p> <p>An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.</p> <p><b>Based on the given information, answer the following questions.</b></p>	
	(i) what is the probability that a new policyholder will have an accident within a year of purchasing a policy?	
	(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?	
	<p><b>Solution:</b> Let <math>E_1</math> = The policy holder is accident prone.  <math>E_2</math> = The policy holder is not accident prone.  <math>E</math> = The new policy holder has an accident within a year of purchasing a policy.</p> <p>(i) <math>P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)</math>  <math>= \frac{20}{100} \times \frac{6}{10} + \frac{80}{100} \times \frac{2}{10} = \frac{7}{25}</math></p> <p>(ii) By Bayes' Theorem, <math>P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)}</math>  <math>= \frac{\frac{20}{100} \times \frac{6}{10}}{\frac{7}{25}} = \frac{3}{7}</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>

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